PURE MATHEMATICS SEMINAR

TROGONOMETRY

Pythagoras theorem

1. Solve the following equations, giving the values of θ from $0°$ to $360°$ inclusive.

a. $3 - 3cos\theta = 2sin^2\theta$

b. $cosec^2\theta = 3tan\theta - 1$

c. $2cot^2\theta + 8 = 7cosec\theta$

2. Prove the following identities

i. $$tan\theta + cot\theta = \frac{1}{sin\theta cos\theta}$$

ii. $$sec^2\theta - cosec^2\theta = tan^2\theta - cot^2\theta$$

iii. $$(sec\theta + tan\theta)(sec\theta - tan\theta) = 1$$

iv. $$sec^2\theta - cosec^2\theta = \frac{sin^2\theta - cos^2\theta}{cos^4\theta sin^4\theta}$$

v. $$\frac{1 - cos^2\theta}{sec^2\theta - 1} = 1 - sin^2\theta$$

3. Eliminate θ from the following equations

$i.\quad x = acos\theta, y = bsin\theta$

$ii.\ \ x = atan\theta, y = bcos\theta$

$iii.\ x = asec\theta, y = b + dcos\theta$

$iv.\ x = cos\theta, y = cosec\theta - cot\theta$

The formula for $\sin(A \pm B)$, $\cos(A \pm B)$

1. Given that $sinA = \frac{4}{5}$ and $cosB = \frac{12}{13}$, where A is obtuse and B is acute, find the values of;

a. $\sin(A + B)$

b. $\cos(A + B)$

c. $\cot(A + B)$

d. $\sin(A - B)$

2. If $\tan(A+B)=\frac{1}{7}$ and $tanA=3$, find the value of $\tan B$

3. If $\sin(x-\alpha)=\cos(x-\alpha)$, prove that $\tan x$

4. Prove the following

a. $\sin(A+B)+\sin(A-B)=2\sin A\sin B$

b. $\cos(A+B)-\cos(A-B)=-2\sin A\sin B$

c. $\tan A+\tan B=\frac{\sin(A+B)}{\cos AcosB}$

5. Prove that

$$\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan A \tan C - \tan B \tan C}$$

Hence prove that if A, B, C are angles of a triangle, then

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

The double angle formula

1. Prove that $\sin 3A = 3\sin A - 4\sin^3 A$

2. Prove that $\cos 3A = 4cos^3 A - 3\cos A$

3. Solve the following equations for the values of θ from $0°$ to $360°$

a. $3\cos 2\theta - \sin\theta + 2 = 0$

b. $\sin 2\theta \cos\theta + sin^2\theta$

c. $3\cot 2\theta + cot\theta = 1$

4. Eliminate θ from the following equations

a. $x = \cos\theta, y = \cos 2\theta$

b. $x = 2\sin\theta, y = 3\cos 2\theta$

c. $x = \tan\theta\,, y = \tan 2\theta$

d. $x = 2\sec\theta, y = \cos 2\theta$

5. Prove that

a. $\frac{\cos 2\theta}{\cos A+\sin A} = \cos A - \sin A$

b. $\frac{\cos A}{\sin B} - \frac{\sin A}{\cos B} = \frac{2\cos(A+B)}{\sin 2B}$

c. $\cot A - \tan A = 2\cot 2A$

d. $\frac{\sin 2A}{1+\cos 2A} = \tan A = \frac{1-\cos 2A}{\sin 2A}$

Section D: The t-Formulae

Solve the following equations for $0° \leq \theta \leq 360°$

a. $2\cos\theta + 3\sin\theta = -2$

b. $7\cos\theta + \sin\theta - 5 = 0$

c. $3\cos\theta - 4\sin\theta + 1 = 0$

d. $3\cos\theta + 4\sin\theta = 2$

Section E: the form $\mathbf{a} \cos \theta + b \sin \theta$

1. Show that $\sqrt{3} \cos \theta - \sin \theta$ maybe written as $2 \cos(\theta + 30°)$ or $2 \sin(60° - \theta)$. find the maximum and minimum values of the expression, state the values of θ between $0°$ and $360°$ for which they occur.
2. Show that $3 \cos \theta + 2 \sin \theta$ maybe written in the form $R \cos(\theta - \alpha)$; where α is acute. Find the values of$R$ and α

GENERAL QUESTIONS

1. Solve the following equations for the values of x from $0°$ to $360°$.

$a.$ $\cos x + \cos 5x = 0$

$b.$ $\sin 3x - \sin x = 0$

$c.$ $\sin(x + 10°) + \sin x = 0$

$d.$ $\cos(2x + 10°) + \cos(2x - 10°) = 0$

2. Prove that $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3tan^2 A}$

3. Prove that $\frac{\sin B - \sin C}{\sin B + \sin C} = \cot\frac{B+C}{2}\tan\frac{B-C}{2}$

4. If $2A + B = 45$, show that $\tan B = \frac{1 - 2\tan A - tan^2 A}{1 + 2\tan A - tan^2 A}$

5. If A, B, and C are angles of a triangle, prove that

$$\cos A + \cos B + \cos C = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$